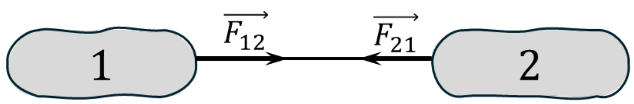
Dynamics

***Df*:** *A section of mechanics that studies mechanical motion based on force concepts*

***Df*:** *Force is a vector physical quantity that characterizes the direction and intensity of interaction between bodies*

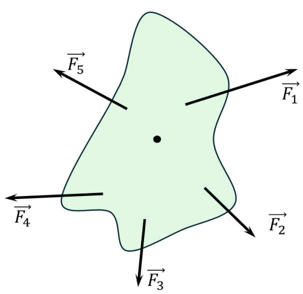
The dimension of force is $[\vec{F}] = \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$



Newton's Laws

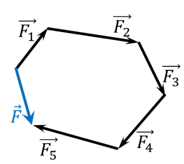
***Lw 1*:** *There are such reference systems relative to which material point moves uniformly and rectilinearly if it is not affected by other bodies or their effects are compensated*

***Lw 2*:** *The acceleration of the center of mass of a rigid body is directly proportional to the resultant of all forces and inversely proportional to its mass.*



Resultant force:

$$\boxed{\vec{F} = \vec{F} + \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_5 = \sum_{i=1}^{5} \vec{F}_i} \quad (1)$$



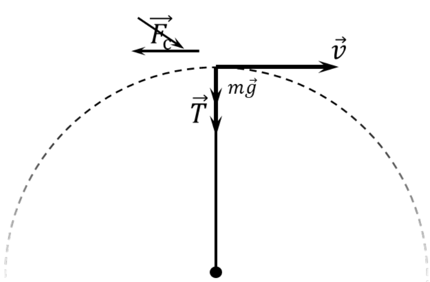
$$|\vec{a}| = |\vec{F}|$$

$$a \sim \frac{1}{m}$$

$$\vec{a} \uparrow\uparrow \vec{F}$$

$$\vec{a} = \frac{\vec{F}}{m} = \frac{\sum_{i=1}^{n} \vec{F}_i}{m} \Leftrightarrow \boxed{m\vec{a} = \sum_{i=1}^{n} \vec{F}_i} \quad (2)$$

***Ex*:**



$$m\vec{a} = m\vec{g} + \vec{T}$$

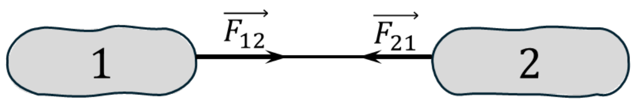
$$T = m(a_n - g) = m\left(\frac{v^2}{R} - g\right)$$

$$\frac{v^2}{l} = g$$

$$\boxed{v_{\min} = \sqrt{gl}} \quad (3)$$

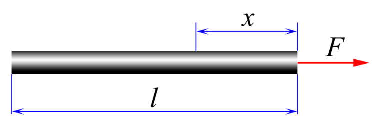
Lw 3*: *Bodies (material points) act on each other, equal in magnitude and opposite in direction.

- F_{12} and F_{21} arise and disappear simultaneously
- F_{12} and F_{21} have the same nature
- Forces cannot be added together because they are applied to different bodies



$$\boxed{\vec{F}_{12} = \vec{F}_{21}} \quad (4)$$

Ex*: *2.1.5. *What force acts in the cross-section of a uniform rod of length l at a distance x from the end to which a force F is applied along the rod?*



For problem 2.1.5

Solution. If we consider a rod of mass m as a single whole, then it will move with acceleration

$$a = \frac{F}{m}$$

Since the rod is inextensible, the acceleration of all its parts is the same and equal to a

Let's consider a small section of the rod of length Δx and mass Δm. Since the rod is homogeneous

$$\Delta m = m\frac{\Delta x}{l}$$

Let's write down Newton's second law for this section.

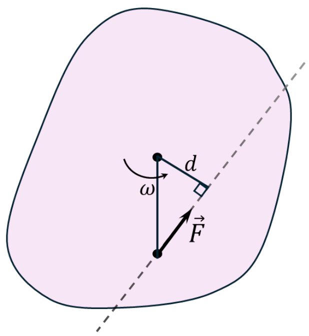
$$a\,\Delta m = F(x + \Delta x) - F(x) \ (1)$$

Where $F(x + \Delta x)$ and $F(x)$ are the interaction forces together with neighbors Let's sum up the expression (1) along the horizontal coordinate from x to l:

$$\sum am\frac{\Delta x}{l} = \sum \Delta F$$

$$F(x) = ma\frac{l-x}{l} \Leftrightarrow \boxed{F(x) = F\left(1 - \frac{x}{l}\right)} \quad (5)$$

Dynamics of rotational motion



$\vec{F}$ — force d — force lever

$$M \pm F \cdot d = \pm F \cdot r \cdot \sin\alpha$$

Dimension of moment of force $[M] = \mathrm{N} \cdot \mathrm{m} = \frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}^2}$

$$\boxed{\vec{M} = \vec{r} \times \vec{F} = -\vec{F} \times \vec{r}} \quad (1)$$

$$\vec{F} = m\vec{a}$$

$$m\vec{a} = \sum_{i=1}^{n} \vec{F}_i$$

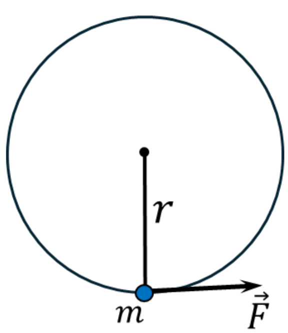
***Df*:** *The moment of inertia* (J) *is a scalar physical quantity characterizing the inertial properties of a body during rotational motion*

The dimension of the moment of inertia is $[J] = \mathrm{kg} \cdot \mathrm{m}^2$

***Lw*:** *The product of the moment of inertia and the angular acceleration of a body is equal to the sum of the moments of forces acting on the body*

$$\boxed{J\beta = \sum_{i=1}^{k} \pm M_i} \quad (2)$$

***Ex*:** *Material point*



Newton's Second Law

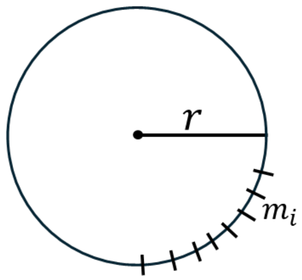
$$ma = F$$

$$mar = F \cdot r$$

$$a = \beta \cdot r$$

$$(mr^2)\beta = M \Rightarrow \boxed{J = mr^2} \quad (3)$$

***Ex*:** *Ring*



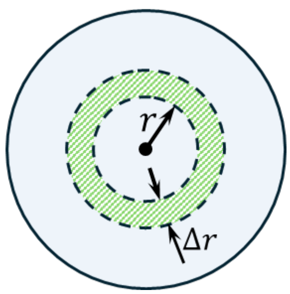
***NO*:** *Moment of Inertia Additive*

$$\boxed{J = J_1 + J_2 + \cdots + J_n} \quad (4)$$

$$J = \sum_{i=1}^{\infty} J_i$$

$$J = \sum_{i=1}^{\infty} m_i r^2 = r^2 \sum_{i=1}^{\infty} m_i = mr^2$$

***Ex*:** *Homogeneous disk*



Surface density of the disk

$$\sigma = \frac{\Delta m}{\Delta S} = \frac{dm}{dS} = \text{const}$$

$$J_i = m_i r_i^2$$

$$\Delta S_i = 2\pi r_i \Delta r_i$$

$$m_i = \sigma \Delta S_i = \sigma 2\pi r_i \Delta r_i$$

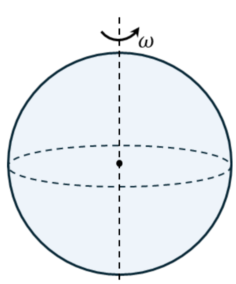
$$J_i = m_i r^2 = 2\pi\sigma r_i^3 \Delta r_i$$

$$J = \sum_{i=1}^{\infty} m_i r^2 = \sum_{i=1}^{\infty} 2\pi\sigma r_i^3 \Delta r_i$$

$$J = 2\pi\sigma \sum_{i=1}^{\infty} r_i^3 \Delta r_i = 2\pi\sigma \frac{r^4}{4} = \frac{\sigma\pi r^4}{2} \Rightarrow \boxed{J = \frac{mR^2}{2}} \quad (5)$$

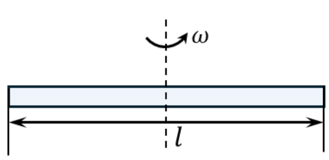
Other examples:

***Ex*:** *Ball*



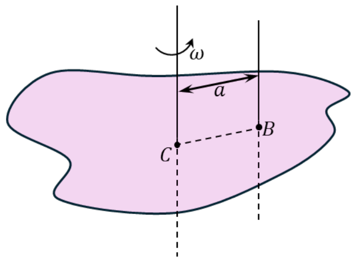
$$\boxed{J = \frac{2mR^2}{5}} \quad (6)$$

***Ex*:** *Homogeneous rod*



$$\boxed{J = \frac{ml^2}{12}} \quad (7)$$

Th*: *Parallel axis theorem

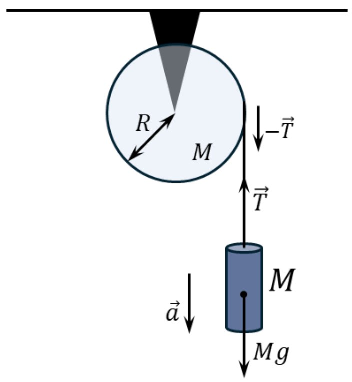


C — center of mass
J_0 — moment of inertia relative to the center of mass

$$J(a) = J_B = J_0 + ma^2$$

$$\boxed{J = J_0 + ma^2} \quad (8)$$

Ex*: *Load on block



Moment of inertia of the block:

$$J = \frac{mR^2}{2}$$

Cargo:

$$Ma = Mg - T$$

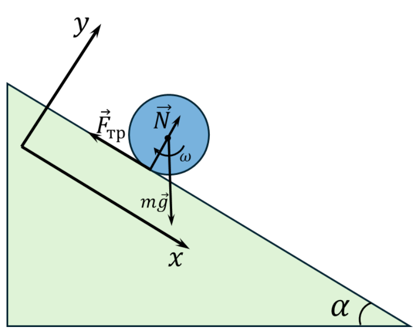
Block:

$$J\beta = \sum_{i=1}^{k} \pm M_i = T \cdot R$$

$$Ja = TR^2 \Rightarrow T = \frac{Ja}{R^2}$$

$$\boxed{a = \frac{Mg}{M + \frac{J}{R^2}} = \frac{g}{1 + \frac{m}{2M}}} \quad (9)$$

Ex*: *Rolling down an inclined plane



F_{fr} — static friction force

$$a = g\sin\alpha \quad (M = 0)$$

$$a = g(\sin\alpha - M\cos\alpha)$$

Newton's second law:

$$ma = mg\sin\alpha - F_{\text{fr}} \quad (OX)$$

$$mg\cos\alpha = N \quad (OY)$$

$$J\beta = F_{\text{fr}}R \Leftrightarrow F_{\text{fr}} = \frac{Ja}{R^2}$$

$$ma = mg\sin\alpha - \frac{Ja}{R^2} \Rightarrow \boxed{a = \frac{g\sin\alpha}{1 + \frac{J}{mR^2}}} \quad (10)$$

Momentum of a body. Law of conservation of momentum

***Df*:** *The momentum of a material point is a vector physical quantity equal to*

$$\boxed{\vec{p} = m\vec{v}} \quad (1)$$

Dimension of impulse $[p] = \text{kg} \cdot \frac{\text{m}}{\text{c}}$

***NO*:** *A. Einstein (Special relativity)*

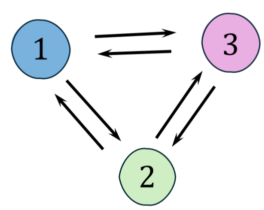
$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}$$

***Df*:** *Impulse of force:* $\vec{F}\Delta t$ Newton's second law in impulse form

$$\boxed{\begin{gathered} \vec{p}_2 - \vec{p}_1 = \vec{F}\Delta t \\ \vec{\Delta p} = \vec{F}\Delta t \\ \vec{F} = \frac{d\vec{p}}{dt} \end{gathered}} \quad (4)$$

The change in the momentum of a system is equal to the momentum of the force acting on it.

***Df*:** *A closed system of bodies is a set of arbitrary objects that interact only with each other*



$$\sum_i \sum_j \vec{F}_{i,j}\Delta t_{i,j} = \vec{0}$$

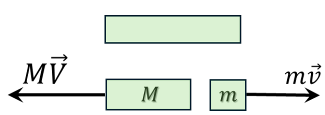
$$\Delta\vec{p} = \vec{F}\Delta = \vec{0}$$

$$\vec{p}_1 = \vec{p}_2 = \vec{const}$$

***Lw*:** The momentum of any closed mechanical system does not change with any interactions within it

$$m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots = m_1\vec{u}_1 + m_2\vec{u}_2 + \cdots$$

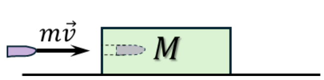
***Ex*:** *The phenomenon of recoil*



$$M\vec{V} + m\vec{v} = \vec{0}$$

$$\vec{V} = -\frac{m\vec{v}}{M}$$

***Ex*:** *Inelastic Impact*



Law of conservation of momentum

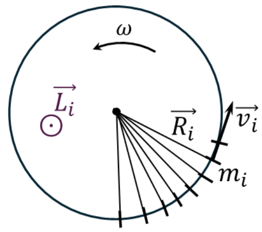
$$m\vec{v} = (m + M)\vec{V}$$

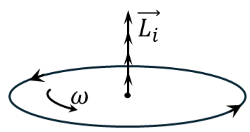
$$\boxed{\vec{V} = \frac{m}{m + M}\vec{v}} \quad (7)$$

Law of conservation of energy

$$\frac{mv^2}{2} = \frac{(m + M)V^2}{2} + Q = \frac{mv^2}{2}$$

Angular momentum of a body. Law of conservation of angular momentum





Df*: *Vector equal to the vector product of the impulse arm and the impulse vector itself

$$\boxed{L = \vec{r} \times \vec{p}} \quad (1)$$

$$\vec{L}_i = R\vec{m}_i \omega R$$

$$L = \sum_{i=1}^{\infty} L_i = \sum_{i=1}^{\infty} m_i \omega R^2 = \omega R^2 \sum_{i=1}^{\infty} m_i = (mR^2)\omega = J\omega$$

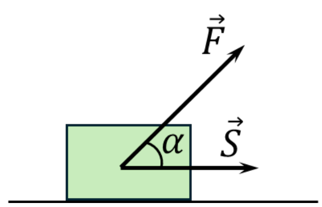
$$\vec{L} = \sum_{i=1}^{\infty} \vec{r_i} \times (m_i \vec{v_i}) = \sum_{i=1}^{\infty} \vec{r_i} \times (m_i \vec{\omega} \times \vec{r_i}) = \sum_{i=1}^{\infty} m_i (\vec{r_i} \times (\vec{\omega} \times \vec{r_i})) = \sum_{i=1}^{\infty} m_i (\vec{\omega} r_i^2 - \vec{r_i}(\vec{\omega} \cdot \vec{r_i})) = \vec{\omega} \sum_{i=1}^{\infty} m_i r_i^2 = \boxed{J\vec{\omega}}$$

$$\boxed{\begin{gathered} \vec{L} = J\vec{\omega} \\ \vec{L} = \vec{r} \times (m\vec{v}) \end{gathered}} \quad (2)$$

***NO*:** *Kepler's second law is the law of conservation of angular momentum*

$$\boxed{\begin{gathered} m_1 v_1 = m_2 v_2 \\ J_1 \omega_1 = J_2 \omega_2 \end{gathered}} \quad (3)$$

Law of conservation of energy



***Df*:** *Work of force(A) is a scalar physical quantity equal to*

$$\boxed{A = \vec{F} \cdot \vec{S} = FS\cos\alpha = F_x S} \quad (1)$$

Dimension of work $[A] = \mathrm{N} \cdot \mathrm{m} = 1\mathrm{J}$

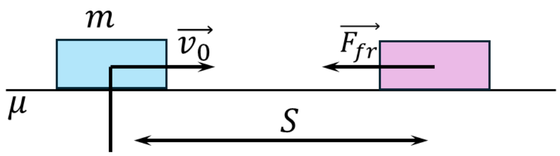
***Df*:** *Kinetic energy is defined by the expression*

$$\boxed{E^k = \frac{mv^2}{2}} \quad (2)$$

***Theorem on the change in kinetic energy*:** *The change in the kinetic energy of a mechanical system is equal to the work of external forces on it*

$$\boxed{\boxed{\Delta E^k = E_2^k - E_1^k = A}} \quad (3)$$

***Ex*:** *Car braking distance*



Kinetic energy change theorem:

$$\Delta E_K = A$$

$$0 - \frac{mv_0^2}{2} = \mu mgS\cos\pi$$

$$\frac{mv_0^2}{2} = \mu mgS \Rightarrow \boxed{S = \frac{v_0^2}{2\mu g}} \quad (4)$$

***Ex*:** *Increment of kinetic energy* ΔE_K*, at* $\Delta v \ll v$

$$\Delta E_K = \frac{m(v+\Delta v)^2}{2} - \frac{mv^2}{2}$$

$$\Delta E_K = \frac{m(2v\Delta v + \Delta v^2)}{2} \approx \frac{2mv\Delta v}{2}$$

$$\boxed{\Delta E_K = mv\Delta v} \quad (5)$$

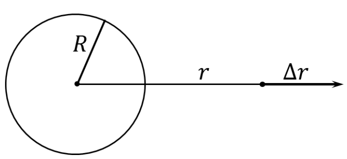
***NO*:** *According to (5), a small increment of kinetic energy is directly proportional to the speed,* $\Delta E^K \sim v$

Df*:** ***Conservative mechanical system *— a mechanical set of bodies interacting without friction and resistance*

***Lw*:** *In the absence of friction and resistance, the sum of the kinetic and potential energy of the system is a constant value for any interactions within the system*

***Heat*:** $mgh_1 + \frac{mv_1^2}{2} = mgh_2 + \frac{mv_2^2}{2} + Q$

***Ex*:** *Greater Heights* ($h \sim R$)



$$F = G\frac{Mm}{r^2}$$

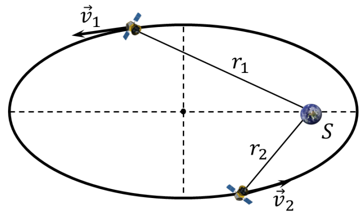
Let us show that E in this case (for astronomical flights)

$$r, r-\Delta r; \Delta r \ll r; \quad \boxed{E = -G\frac{mM}{r}} \quad (6)$$

$$A = FS\cos\alpha = \boxed{G\frac{mM}{r^2}\Delta r}$$

$$A = -\Delta E = E_1 - E_2 = -GmM\left(\frac{1}{r-\Delta r} - \frac{1}{r}\right)$$

$$A = -GmM\frac{\Delta r}{(r-\Delta r)r} \approx \boxed{G\frac{mM}{r^2}\Delta r}$$



$$\frac{mv_1^2}{2} - G\frac{mM}{r_1} = \frac{mv_2^2}{2} - G\frac{mM}{r_2}$$

***First cosmic velocity*:** $v_1 = \sqrt{gR} = 7.9$ km/s

***Second cosmic velocity*:** $v_2 = \sqrt{2G\frac{M}{R}} = \sqrt{2}v_1 = 11.2$ km/s