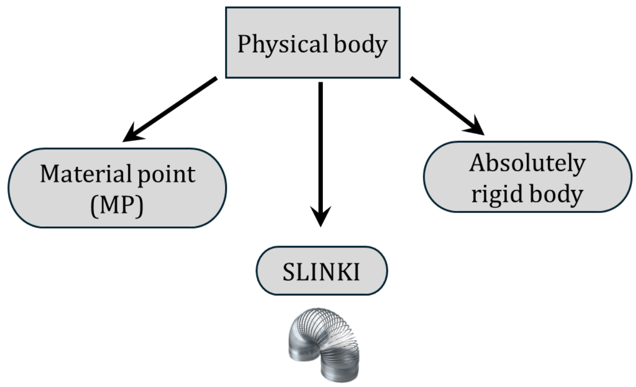
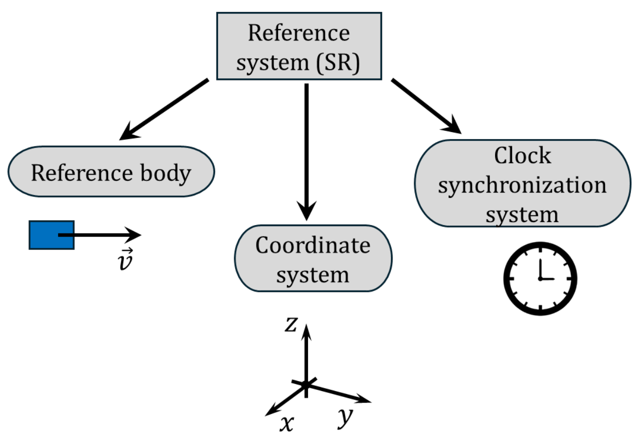
Kinematics

Uniform and uniformly accelerated motion

***Df*:** *Mechanics is a branch of physics that studies the laws of motion and the causes that cause it.*

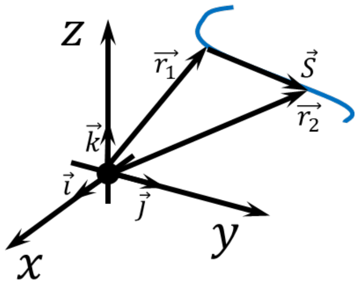


***Re*:** *Reference System (SR)*



***NO*:** *In Newtonian terms: time is absolutely* $(t = t')$*; Einstein:* $(t \neq t')$

***Df*:** *Radius of curvature* $\vec{r}$*—Directed segment connecting the origin of coordinates with the current position of the material point (MP).*



From the drawing:

$$\vec{r_2} = \vec{r_1} + \vec{S}$$

$$\vec{S} = \vec{r_2} - \vec{r_1} = \Delta\vec{r}$$

Vector sum:

$$|\vec{i}| = |\vec{j}| = |\vec{k}| = 1$$

$$\boxed{\vec{r} = x \cdot \vec{i} + y \cdot \vec{j} + z \cdot \vec{k}} \quad (1)$$

From the Pythagorean theorem:

$$\boxed{r = \sqrt{x^2 + y^2 + z^2}} \quad (2)$$

Re*: *Path is a scalar physical quantity equal to the length of the projection of a material point

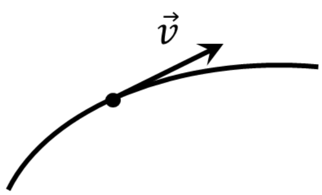
Dimension of the path $[S]$ =m

Re*: *Displacement is a directed segment connecting the start point and the end point of the point's movement

Rectilinear unidirectional motion

$$\boxed{|\vec{S}| = l} \quad (3)$$

Df*: *Instantaneous velocity of a material point:

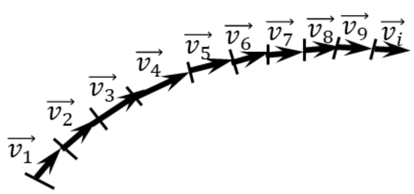


In a very short period of time $\Delta t \to 0$

$$\boxed{\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}} \quad (4)$$

$$\Delta\vec{r} = \vec{v}\Delta t$$

The dimension of speed is $[v]$ =m/s



Newton's method split into N parts

$$\Delta\vec{r}_1 = \vec{v}_1\Delta t_1$$

$$\Delta\vec{r}_2 = \vec{v}_2\Delta t_2$$

$$\vdots$$

$$\Delta\vec{r}_i = \vec{v}_i\Delta t_i$$

The displacement $\vec{S}$ is found as the vector sum of displacements over small intervals Δt

$$\vec{S} = \vec{r}_1 + \vec{r}_2 + \cdots + \vec{r}_n = \sum_{i=1}^{\infty} \vec{v}_i\Delta t_i$$

$$\vec{S} = \vec{v}_1\Delta t_1 + \vec{v}_2\Delta t_2 + \cdots + \vec{v}_n\Delta t_n$$

$$\boxed{S = \sum_{i=1}^{\infty} \vec{v}_i\Delta t_i} \quad (5)$$

Considering the rectilinear motion

$$\vec{v}_i = \vec{v} = \text{const}$$

$$\vec{v}\sum_{i=1}^{\infty} \Delta t_i = \vec{v}t$$

$$\boxed{\vec{S} = \vec{v}t} \quad (6)$$

***Df*:** *Acceleration of a material point is a vector physical quantity equal to the change in velocity over a small period of time* Δt

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \vec{v}'(t)$$

The dimension of acceleration is $[a] = \text{m/s}^2$

***Df*:** *With uniformly accelerated motion, the velocity of a material point changes by the same amount over any equal time intervals*

$$\Delta\vec{v} = \vec{a}\Delta t$$

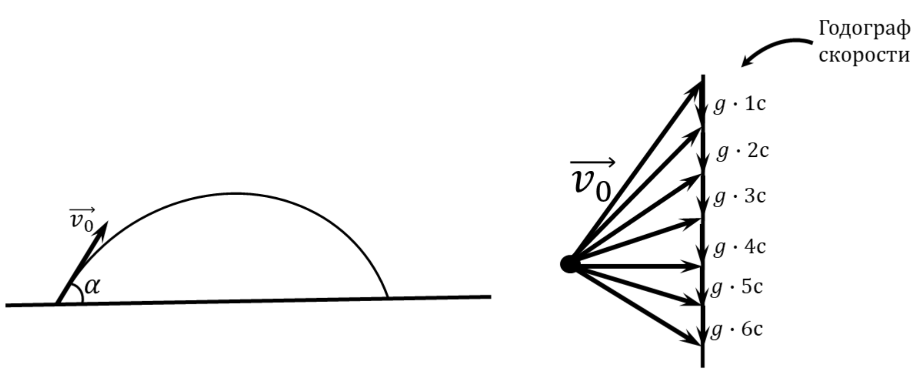
$$\vec{v} - \vec{v}_0 = \vec{a}\Delta t$$

$$\boxed{\vec{v} = \vec{v}_0 + \vec{a}\Delta t} \quad (7)$$

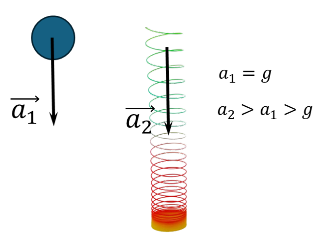
$$\boxed{\vec{S} = \vec{v}_0 t + \frac{\vec{a}t^2}{2}} \quad (8)$$

$$S = \frac{v^2 - v_0^2}{2a}; \quad S = \frac{v + v_0}{2}t$$

***Ex*:**



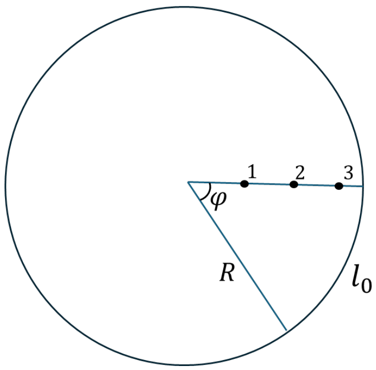
***Dem*:**



"Super free fall"

Rotational motion

***Df*:** *The trajectory of a material point is a circle of radius R*



$$l_1 < l_2 < l_3$$

Rotation angle φ radians

$$\varphi_1 = \varphi_2 = \varphi_3$$

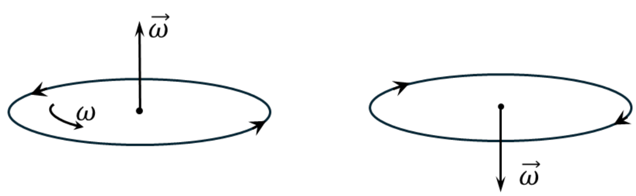
$$\boxed{\varphi = \frac{l_0}{R}; \quad l_0 = \varphi R} \quad (1)$$

***Df*:** *Angular velocity of a material point:*

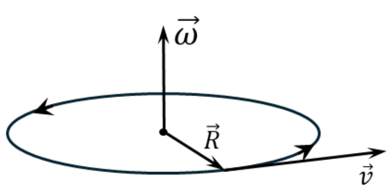
$$\boxed{\omega = \frac{\Delta\varphi}{\Delta t} = \frac{d\varphi}{dt}} \quad (2)$$

The dimension of angular velocity is $[\omega]$ =s^{-1}

***NO*:** *$\vec{\omega}$ is a vector quantity*



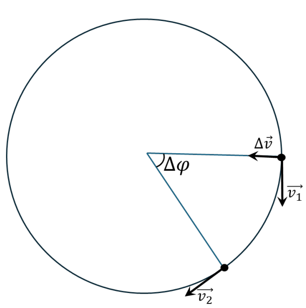
Right Screw Rule



$$v = \frac{\Delta l}{\Delta t} = \frac{R\Delta\varphi}{\Delta t} = \omega R$$

$$\boxed{\vec{v} = \vec{\omega} \times \vec{R}} \quad (3)$$

***Ex*:** *Centripetal acceleration: $a = \frac{v\Delta\varphi}{\Delta t} = vR$*



$$a = \frac{\Delta v}{\Delta t} = \{\Delta v = u\Delta\varphi\}$$

***Ex*:**

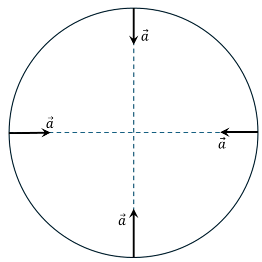
$$2\pi R = vt \qquad T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$$

$$2\pi = \omega t \qquad \boxed{\omega = \frac{v}{R} \quad v = \omega R} \quad (4)$$

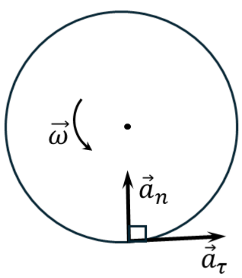
$$\omega = 2\pi\nu$$

***NO*:** *Formula for calculating centropetral acceleration a*

$$\boxed{a = v \cdot \omega = \omega^2 R = \frac{v^2}{R} = 4\pi^2\mu^2 R = \frac{4\pi^2 R}{T^2}} \quad (5)$$

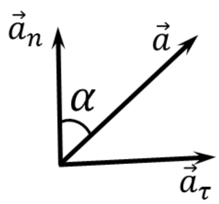


***Ex*:** *Uneven circular motion: (Tangential acceleration)*



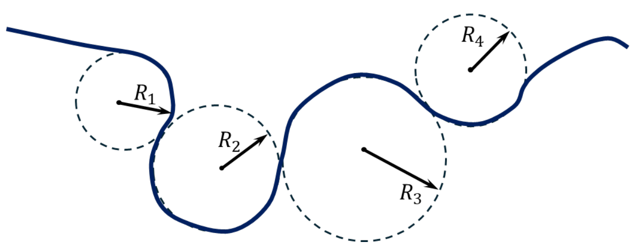
$$\boxed{\begin{cases} a_\tau = \frac{\Delta v}{\Delta t} \\ v = v_0 + a_\tau t \\ \breve{l} = v_0 t + \frac{a_\tau t^2}{2} \end{cases}} \quad (6)$$

***Df*:** *Total acceleration of a material point*



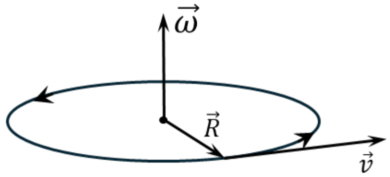
$$\boxed{\begin{cases} \vec{a} = \vec{a_n} + \vec{a_\tau} \\ a = \sqrt{a_n^2 + a_\tau^2} \\ \tan\alpha = \frac{a_\tau}{a_n} \end{cases}} \quad (7)$$

***NO*:** *Newton's Method:*



An arbitrary trajectory is divided into different arcs of a circle (R_i)

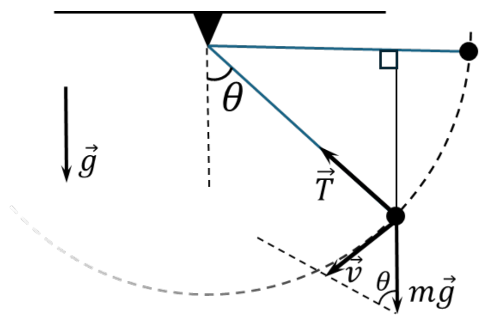
***Ex*:** $a_n = a_{centr} = v\omega$



From the drawing:

$$\boxed{\vec{a}_n = \vec{a}_{\text{centr}} = \vec{\omega} \times (\vec{\omega} \times \vec{R})} \quad (8)$$

***Ex*:** *"Horizontal Swing"*



$$a(\theta)-?$$

Newton's Second Law

$$m\vec{a} = m\vec{g} + \vec{T} + \vec{F}_c \quad (F_c \ll mg)$$

$$m(\vec{a}_n + \vec{a}_\tau) = mg\sin\theta$$

$$\boxed{a_\tau = g\sin\alpha} \quad (9)$$

Law of conservation of energy

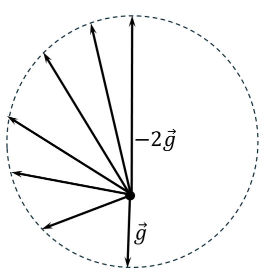
$$mgh = \frac{mv^2}{2}$$

$$gl\sin\theta = \frac{v^2}{2}$$

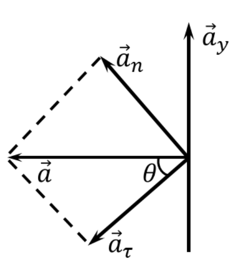
$$\boxed{a_n = \frac{v^2}{R} = 2g\cos\theta} \quad (10)$$

$$a = \sqrt{a_n^2 + a_\tau^2} = \sqrt{g^2\sin^2\alpha + 4g^2\cos^2\alpha} = \boxed{g\sqrt{1 + 3\cos^2\alpha}} \quad (11)$$

***Df*:** *Hodograph of acceleration of a material point (set of end points of a vector)*



***NO*:** *At the moment of time when the vector of total acceleration is horizontal, its projection on the vertical axis is equal to O*



$$a_n\cos\alpha = a_\tau\sin\alpha$$

$$2g\cos^2\alpha = g\sin^2\alpha$$

$$\boxed{\theta = \arctan\sqrt{2} \approx 55°} \quad (12)$$

Df**:** *Angular acceleration of a material point* (β, ε)

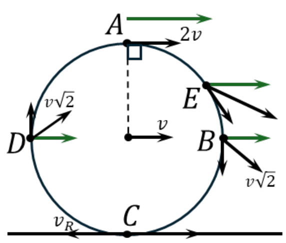
$$\boxed{\beta = \frac{\Delta\omega}{\Delta t}} \quad (13)$$

The dimension of angular acceleration is $[\omega] = \mathrm{s}^{-2}$ With $\beta = \mathrm{const}$

$$\boxed{\begin{aligned} &\omega = \omega_0 + \beta t \\ &\varphi = \varphi_0 + \omega_0 t + \frac{\beta t^2}{2} \end{aligned}}$$

$$v = \omega R$$
$$\frac{\Delta v}{\Delta t} = \frac{\Delta \omega R}{\Delta t}$$
$$\beta = \frac{a_\tau}{R} \quad (14)$$

Ex**:** *Rolling without slipping*



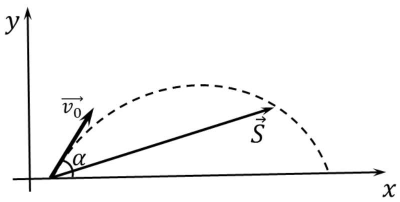
$\vec{v}$ — translational motion velocity
$\vec{v}_R$ — rotational motion velocity

$$\boxed{\vec{V} = \vec{v} + \vec{v}_R} \quad (15)$$

C — instantaneous center of rotation
$v_c = 0 \Rightarrow$ without slipping

$$\boxed{v = \omega R; \quad \omega = \frac{v}{R}}$$

Movement at an angle to the horizon



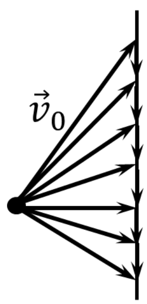
Newton's Second Law:

$$m\vec{a} = m\vec{g} + \vec{T} + \vec{F_c} \quad (F_c \ll mg)$$

$$\vec{a} = \vec{g} = \vec{\text{const}}$$

$$\vec{v}(t) = \vec{v}_0 + \vec{g}t$$

Speed hodograph:



$$\boxed{t_n = 2t_1 = \frac{2v_0 \sin\alpha}{g}} \quad (1)$$

$$\begin{cases} v_x(t) = v_0 \cos\alpha \\ v_y(t) = v_0 \sin\alpha - gt \end{cases}$$

$$0 = v_0 \sin\alpha - gt$$

$$\boxed{t_{\text{flight}} = \frac{v_0 \sin\alpha}{g}} \quad (2)$$

$$\vec{S} = \vec{v}_0 t - \frac{gt^2}{2}$$

$$\boxed{\begin{cases} x(t) = v_0 \cos\alpha \cdot t \\ y(t) = v_0 \sin\alpha \cdot t - \frac{gt^2}{2} \end{cases}} \quad (3)$$

Equation of trajectory:

$$t = \frac{x}{v_0 \cos\alpha}$$

$$\boxed{y(x) = x \tan\alpha - \frac{g \cdot x^2}{2 \cdot v_0^2 \cos^2\alpha}} \quad (4)$$

Flight range:

$$S = v_0 \cos\alpha \cdot \frac{2v_0 \sin\alpha}{g} = \frac{v_0^2}{g}(2 \sin\alpha \cos\alpha)$$

$$\boxed{S = \frac{v_0^2}{g} \sin 2\alpha} \quad (5)$$